

Cosmic Superstring Gravitational Lensing Phenomena: Predictions for Networks of (p,q) Strings

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Abstract

The unique, conical spacetime created by cosmic strings brings about distinctive gravitational lensing phenomena. The variety of these distinctive phenomena is increased when the strings have non-trivial mutual interactions. In particular, when strings bind and create junctions, rather than intercommute, the resulting configurations can lead to novel gravitational lensing patterns. In this brief note, we use exact solutions to characterize these phenomena, the detection of which would be strong evidence for the existence of complex cosmic string networks of the kind predicted by string theory-motivated cosmic string models. We also correct some common errors in the lensing phenomenology of straight cosmic strings.

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I. INTRODUCTION

Some time after the existence of cosmic strings was proposed [1], several researchers recognized that the conical spacetime generated by cosmic strings leads to a unique gravitational lensing signature: undistorted double images [2]; the discovery of even a single such gravitational lensing event would be seen as irrefutable evidence for the existence of cosmic strings. Previous detailed studies of string lensing phenomena have focused on infinite strings and string loops, whether straight or wiggly [3]. For standard, abelian Higgs strings, these are the only lensing effects that one would expect to find.

Recently, there has been a renaissance of interest in cosmic strings. This renewal was begun by the recognition that cosmic strings are copiously produced in the aftermath of the brane collision that generates reheating in brane-world models of inflation in string theory [4]. In addition to reviving interest in cosmic strings, these new studies have enriched cosmic string phenomenology by proposing the existence of two basic cosmic string types: Fundamental, or F-strings, and one-dimensional Dirichlet-brane strings, or D-strings. These two kinds of strings are able mutually to interact to form bound states. These bound states are known as (p, q) strings, as they are composed of p F-strings and q D-strings [5]. String binding allows for a variety of new observational phenomena, yet does not cause any cosmological catastrophes [6]. In particular, the existence of string binding interactions generically implies that there will be Y -shaped junctions of three strings that form each time there is a string binding event. These Y -shaped junctions give rise to lensing phenomena that are qualitatively distinct from anything standard abelian Higgs models can produce.

Since cosmic strings are perhaps the only directly observable remnant of brane inflation, it is vital that we identify any distinctive effects that are peculiar to the cosmic “superstrings” that are produced in brane-collision reheating. Cosmic superstrings may be our best hope for directly observing some aspects of string theory. The observation of even a single distinctive lensing event – one that could not be explained in an abelian Higgs model – would be a “smoking gun” for the existence of a non-trivial cosmic string network that is the hallmark of cosmic superstring models. We note, however, that similar junctions are found in non-abelian string networks as well (e.g., the S^3 networks studied in ref. [7, 8]).

In this brief note, we describe the principal novel phenomena arising from the binding junctions that characterize non-trivial networks of cosmic superstrings. In §II, we review lensing by a straight cosmic string, writing down some general formulae that have not previously appeared in

the literature. In Fig.1 we illustrate the quintessential signature of a (p, q) network junction. This is an imaginative construction of what the galaxy NGC 2997 [15] might look like if it were lensed by a Y -shaped string junction. In §III, we derive the simple procedure used to generate this image.

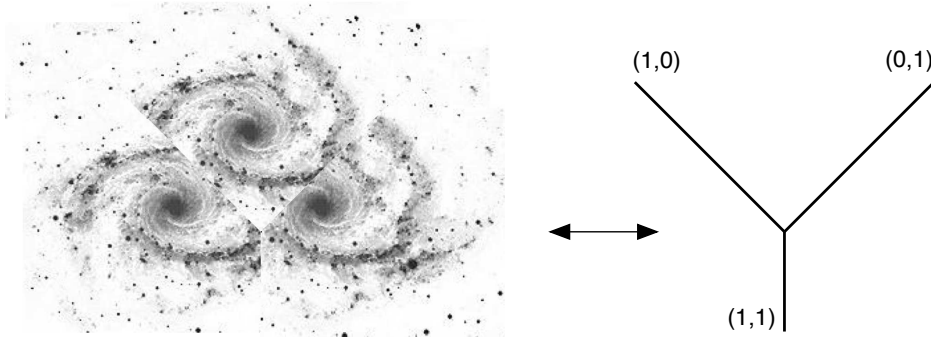


FIG. 1: Illustration of lensing of a single galaxy by a (p, q) network junction. Note that each image is partially obscured, which is a generic feature of cosmic string lensing events [9]. This image is an imaginative construction, not an actual observation.

II. REVIEW OF STRAIGHT STRING LENSING

The lensing due to a straight string is surprisingly simple; this comes about because the surrounding space-time is locally flat. Many details of this lensing can nevertheless be quite subtle, and so here we correct two errors found in the literature, namely the angular separation formula and the orthogonality of the image pairs with respect to the observed cosmic string. A string oriented in a direction $\hat{\mathbf{s}}$ produces two images on opposite sides of the string separated by an “angular separation vector” $\vec{\delta\varphi}$ of magnitude

$$|\vec{\delta\varphi}| = 8\pi G\mu \sqrt{\gamma^2(1 + \hat{\mathbf{n}} \cdot \mathbf{v})^2 - \cos^2\theta} \frac{D_{s,cs}}{D_{s,o}}. \quad (1)$$

In our notation, bold face variables represent 3-vectors, bold hatted variables are unit 3-vectors, and over-arrows signify 2-vectors which live in the plane orthogonal to the direction of sight. Here $G\mu$ is the dimensionless string tension, $\gamma = 1/\sqrt{1 - \mathbf{v}^2}$, \mathbf{v} is the string velocity, $\hat{\mathbf{n}}$ is the unit vector directed along the line of sight, θ is the angle between the cosmic string and $\hat{\mathbf{n}}$ (i.e., it is defined by $\hat{\mathbf{n}} \cdot \hat{\mathbf{s}} = \cos\theta$), $D_{s,cs}$ is the distance from the source to the cosmic string, and $D_{s,o}$ is the distance from the source to the observer. Because cosmic strings are boost invariant along their axis, we will always work in the gauge where \mathbf{v} satisfies $\mathbf{v} \cdot \hat{\mathbf{s}} = 0$. Vilenkin [2] pointed out that a straight cosmic

string in motion will appear curved, like a large hyperbola in the sky; we note that, as expected, the apparent vanishing point of the hyperbola corresponds to the point where the magnitude of the angular separation vector goes to zero ($\delta\vec{\varphi} \rightarrow 0$). The reason for the apparent curvature is because what we see is the cosmic string world sheet intersected with our past light cone, which we mentally project onto the $t = 0$ hyper-plane. The cosmic string equations of motion are solved by

$$\mathbf{x}(\sigma, t) = \hat{\mathbf{s}}\sigma/\gamma + \mathbf{v}t + \mathbf{b} \quad (2)$$

with impact parameter \mathbf{b} (with respect to the origin/observer) orthogonal to both $\hat{\mathbf{s}}$ and \mathbf{v} . The *observed* cosmic string is described by the illusory embedding

$$\mathbf{y}(\sigma, t) = \mathbf{x}(\sigma, t - |\mathbf{y}(\sigma, t)|) \quad (3)$$

and so solving for \mathbf{y} yields

$$\mathbf{y}(\sigma, t) = \hat{\mathbf{s}}\sigma/\gamma + \mathbf{v}(\gamma^2 t - \sqrt{\mathbf{b}^2\gamma^2 + \sigma^2 + \mathbf{v}^2\gamma^4 t^2}) + \mathbf{b}. \quad (4)$$

The apparent string then has orientation given by $\hat{\mathbf{s}}_{\text{apparent}} = \mathbf{y}'(\sigma, t)/|\mathbf{y}'|$, which is in general not equal to $\hat{\mathbf{s}}$. These string orientation vectors may be pulled back onto the sky via

$$\vec{s} = \hat{\mathbf{s}} - (\hat{\mathbf{n}} \cdot \hat{\mathbf{s}})\hat{\mathbf{n}} \quad (5)$$

$$\vec{s}_{\text{apparent}} = \hat{\mathbf{s}}_{\text{apparent}} - (\hat{\mathbf{n}} \cdot \hat{\mathbf{s}}_{\text{apparent}})\hat{\mathbf{n}}. \quad (6)$$

The difference between apparent and actual string orientation can then be characterized by the angle between these two 2-vectors:

$$\cos \beta = \frac{\vec{s} \cdot \vec{s}_{\text{apparent}}}{|\vec{s}||\vec{s}_{\text{apparent}}|} = \frac{\cos \theta \mathbf{v} \cdot (\hat{\mathbf{n}} \times \hat{\mathbf{s}})\gamma}{\sin \theta \sqrt{\gamma^2(1 + \hat{\mathbf{n}} \cdot \mathbf{v})^2 - \cos^2 \theta}}. \quad (7)$$

Interestingly, the relative angle between $\delta\vec{\varphi}$ and \vec{s} can be shown to be $\pi/2 + \beta$ which implies that the angular separation vector is always orthogonal to the *apparent* cosmic string as in Fig. 2.

$$\delta\vec{\varphi} \cdot \vec{s}_{\text{apparent}} = 0 \quad (8)$$

One might wonder how it is possible for a cosmic string that appears to be bent in the shape of a hyperbola to produce an undistorted image. In fact, the two images are not completely identical: They are moving at slightly different velocities, which gives rise to both the Kaiser-Stebbins (blue-shift) effect, as well as a relative “hyperbolic distortion” due to the finite travel time of light. So the distortion induced by an extremely relativistic string is none other than the distortion that all moving bodies appear to have; c.f. Eqn. (3).

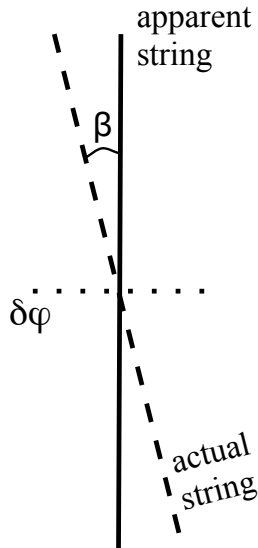


FIG. 2: A schematic view of the lensing due to a single string in the most general case; when the string’s velocity is non-zero, the angular separation vector is perpendicular to the apparent string, rather than to the actual string.

III. LENSING BY Y-JUNCTIONS

A feature of superstring networks is that they are composed of at least two distinct string species: so-called fundamental or F-strings as well as D1-branes, or D-strings. These different types of strings can mutually interact via binding, creating (p, q) bound states composed of p F-strings and q D-strings [5]. The tension of such strings is given by

$$\mu_{p,q} = \sqrt{p^2 \mu_F^2 + q^2 \mu_D^2} \quad (9)$$

Networks of such strings are cosmologically safe, as they are expected readily to go to scaling [6].

It is in the region near the Y -shaped junctions formed after collisions that the new string lensing effects are seen. In Fig. 1, we showed an image as it might actually appear, with the invisible strings and angular separation vectors suppressed. In Fig. 3, we show a mock-up of the triple image formed in the vicinity of a Y -junction together with the strings that produce the image. In brief, each leg of the Y -junction lenses exactly like an infinite straight string.

To motivate the above result, let us begin by considering a source simultaneously imaged by two long strings.

For two overlapping strings, it is straightforward to construct a set of rules for the multiple imaging of a single source. Each straight cosmic string has an associated two dimensional “angular

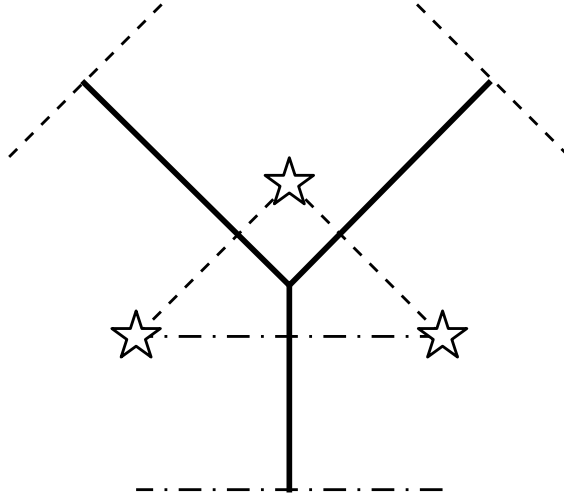


FIG. 3: The imaging pattern of a three-way junction. The dark lines represent the force-balanced string junction; in reality, strings themselves are invisible. The dotted and dot-dashed lines represent, schematically, the angular separation vectors associated with the strings. We suppress the heads of the vectors, since their orientation is arbitrary. For string tensions in the upper range allowed by observations – that is, $G\mu \sim 10^{-7}$ [10] – the size of the angular separation vectors would be of order 1 arc-second. The stars represent the lensed images, with the angular separation vectors drawn in for illustrative purposes.

separation vector” the length of which is given by Eqn. (1). For string tensions in the upper range allowed by observations [10], the magnitudes of these angular separation vectors are of order 1 arc-second. The orientation of the angular separation vector is perpendicular to the associated cosmic string.

1. Begin with an original image (i.e. the object); the choice of which image to begin with is arbitrary.
2. Construct a parallelogram originating at the object and generated by the angular separation vectors associated with each cosmic string; each corner represents a new image.
 - (a) Each image (except the object) will be associated with the set of cosmic strings whose angular separation vectors were used to create it.
 - (b) If exactly those strings that are associated with an image – and no others – lie between the object and that image, then that image will be visible; otherwise, that image will be invisible.
3. Thus, given the existence of one visible image (the object), the location and status – visible or invisible – of the other three images is known. An example of this is shown in Fig. 4.

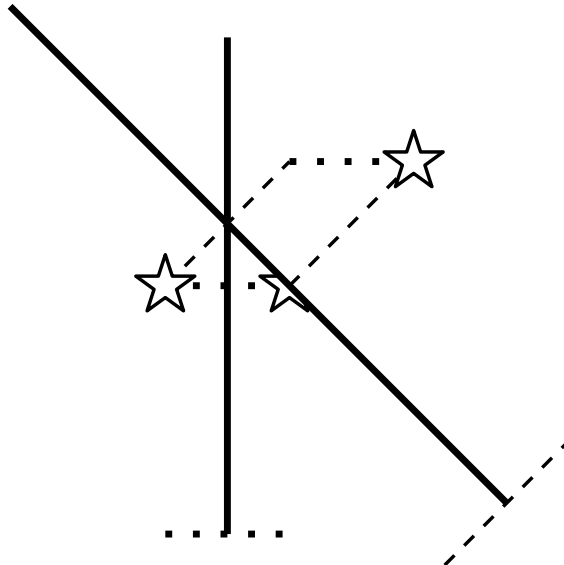


FIG. 4: The imaging pattern for two overlapping cosmic strings. The dotted and dot-dashed lines represent, schematically, the angular separation vector associated with the strings. Notice that only the object and two of the three images are visible.

4. This procedure is consistent with the fact that a visible image is made invisible when and only when any cosmic string moves across it, and an invisible image is made visible only when a cosmic string moves across it.

For three overlapping strings, we follow the same procedure as before, with the same rules. The only difference is that rather than forming a parallelogram, three angular separation vectors lead to a parallelepiped. We show an example of this sort of diagram in Fig. 5.

Finally, let us stipulate that the three strings are coplanar and intersect at one point. If they are to be in force balance, then their orientation unit vectors must obey

$$\sum_{i=1}^3 \mu_i \gamma_i \hat{\mathbf{s}} = 0. \quad (10)$$

The γ -factors correct for the Lorentz contraction caused by boosting in a direction not perpendicular to all three strings.

If the three infinite strings satisfy Eqn. (10) then one may “cut-and-paste” them into two Y-junctions without changing the energy-momentum tensor. One of the Y-junctions can then be pulled away to reveal a single junction and its lensed images.

When cosmic strings intersect at a three-way junction, the background metric is solvable if and only if [11] the strings obey the force balance equation. Dynamically, however, the force balance

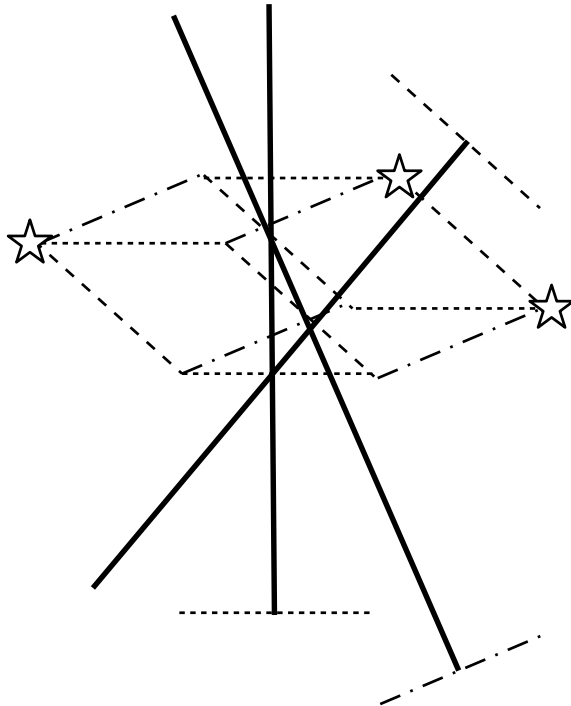


FIG. 5: The imaging pattern for three overlapping cosmic strings. Only the object and two of the seven images are visible.

equation is always satisfied since the neighborhood of the junction point can be taken to have an arbitrarily small mass, thus allowing it to respond instantaneously to any net force. This would not be the case if the vertex had a large mass (e.g., if it were a monopole). Thus there is a consistency check that must be performed for lensing by Y -junctions: each string lenses the two images surrounding it, and so the picture is over determined. In other words, each object has two images which must also satisfy the lensing equation of the string separating them. This means we must check that the angular separation vector triangle closes:

$$\sum_{i=1}^3 \vec{\delta\varphi}_i = 0. \quad (11)$$

In fact, we find that the force balance Eqn. (10) is a sufficient condition for Eqn. (11) to be satisfied. This could be inferred from the first claim of this paragraph, but is also easy to verify directly using the equations found in §II.

IV. CONCLUSION

We have presented an exact solution for both the lensing of three-string junctions and for multiple overlapping strings. We do not yet know how prevalent such junctions would be in

realistic networks of (p,q) strings. Since they are expected to be somewhat rare, however, our chief hope for locating such a junction would be, first, to locate an actively lensing cosmic string. As yet, no astronomical observations have yielded unambiguous evidence for such a lensing event, though there is currently a candidate observation, CSL-1 [12], awaiting confirmation or refutation by the Hubble Space Telescope [13]. In passing, we note that, since high velocities generally enhance lensing (see Eqn. 1), there cannot be a firm lower bound placed on the tension of the inferred string tension from this event, if it proves to be authentic. Should such an event be found, we might hope to track it across the sky by interpolation between further lensing events or through the Kaiser-Stebbins effect [14] until a junction could be located along its length. If it so happens that CSL-1 is confirmed as an authentic cosmic string lensing event, an eager modern-style search would doubtless ensue in hopes of tracking down further such events; in such a scenario, the sort of cross-sky tracking suggested above might become possible. Within any such search, the discovery of even one triple imaging event as described in this paper would be an unmistakable indicator of the existence of a cosmic string network with non-trivial interactions, the very kind predicted in brane inflationary models. Finding this sort of smoking gun – so rare as to be invisible to serendipitous discovery – might be possible in such a scenario, giving us hope for an experimental examination of string theory unthinkable even a few years ago.

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APPENDIX A: FLY'S EYE EFFECT

A surprising secondary effect can be obtained if a large number of Y -junctions are arranged near one another: a single source can be identically imaged many, many times. We include this effect, though it is unlikely to arise in nature – where the junctions tend to speed apart relativistically – as an example of the extreme limit of the Y -lensing phenomenon. We further note that, while unlikely in present models, it has frequently been mentioned in the past [5, 7, 8] that networks of strings that become frozen could have a fly's eye-style structure. We illustrate, on a small scale, what such a configuration could look like in Fig. 6; on a cosmic scale, the number of links and associated images could, of course, be multiplied almost indefinitely.

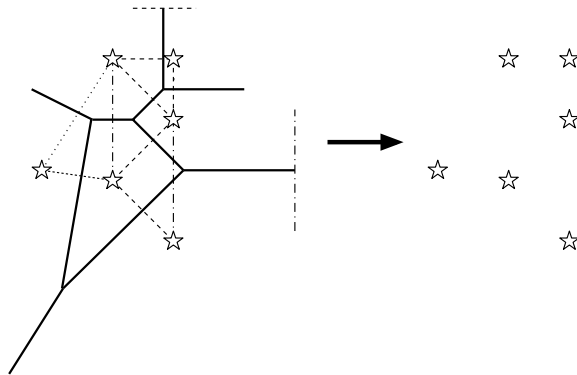


FIG. 6: This set-up of strings and associated images is unlikely ever to occur in nature. It is included for illustrative purposes.